CHAPTER 2

Introduction to GPS

SATELLITE-BASED SYSTEMS

We now describe the difference in technology between a satellite positioning system and a terrestrial positioning system.

Mathematics is important to the deployment and usage of satellites. There is a combination of different types of mathematics. Processing digital data extracted from satellite signals requires deterministic mathematics. These signals pass through the atmosphere and fluid mechanics is used to model that. As many of the calculations are approximations, the mathematics of uncertainty comes into play. As an example of the relevance of mathematics, take Explorer 1 which was the United State’s first satellite, of any sort. It was launched in 1958. It was a rotating satellite, with rotation taking place about a certain axis. Unfortunately, the rotation became unstable and the satellite ended up rotating about the wrong axis. At the time, the mathematics used to analyze the intended rotation, the 200-year-old Euler’s theory, had not predicted the instability. Nevertheless, the satellite was still able to complete its mission, and it discovered the Van Allen radiation belts. However, some satellites are critically dependent on their orientation and thus, if instability occurs, the mission fails. The problem encountered with Explorer 1, was the impetus to find an improved way of assessing the stability of a rotating satellite. These efforts have recently resulted in an extension to Euler’s theory.

Uncertainties in Global Positioning System’s (GPS) positioning: A mathematical discourse concerns itself mainly with GPS as it is the dominant global positioning and navigation satellite-based system. It is known for its versatility. Much of what is written is relevant to other satellite-based positioning systems.

Today, we all know what a GPS receiver is: it communicates with a satellite system and lets you know where you are on a map. A receiver receives signals from several orbiting satellites and processes them. The receiver has a built-in map. A GPS receiver calculates its position on
Earth in three-dimensional coordinates, which can be converted to latitude, longitude, and altitude. A receiver can be used anywhere on Earth, at any time of the day or night, and in any weather conditions. A receiver needs to have line-of-sight with a satellite for it to receive its signal. This means that the route from the receiver to the satellite cannot be obstructed, so it cannot be used under a bridge, for example. Radio waves cannot pass through bridges, buildings, soil, trees, walls, water, and so on.

A GPS receiver used in a vehicle is not suited to some environments, such as indoor parking areas and tunnels. In these environments the receiver does not have lines of sight with the satellites. In street canyons, a receiver may not have lines of sight with the satellites and even if it does the signals could be corrupted due to multipath. To overcome the above problems, it has been suggested that wireless local area networks could be used (WLANs).

A receiver is a passive device, it simply receives signals. An unlimited number of receivers can process GPS signals at the same time without fear of overloading the system. This is analogous to broadcast TV and radio, where millions of people can receive signals without any degradation of the broadcast.

The scientific and technological aspects of satellite-based positioning systems are highly complex; however, a user can make use of a system without knowledge of these aspects.

Prior to GPS, precise positioning was often accomplished using either inertial guidance systems or low-altitude satellites. Global Navigation Satellite Systems (GNSSs) are satellite positioning systems (sat-navs). The acronym GNSSs is in common usage and refers to all past, existing, and planned systems. The first navigation satellites were developed in the late 1950s, and were used to navigate aircraft and ships. In 1958 the US Navy created a satellite navigation system called TRANSIT whose purpose was to update the inertial navigation systems used by nuclear submarines. Other early systems include, SECOR, SIKADA, and TIMATION. Several navigation satellite systems are under development or operational. Some are competitors to GPS and some could augment GPS. Some of the satellite constellations that are currently in use for satellite positioning or under development are:

1. COMPASS of China (also known as Beidou in Chinese). China operates the BeiDou-1 system for regional use. It is an experimental navigation system. Unlike Galileo, GLObal NAvigation Satellite System
(GLONASS), and GPS, BeiDou-1 uses satellites in geostationary orbit. China plans to extend BeiDou-1 to become a GNSS (Beidou 2).

2. GALILEO of the European Union (EU), operated by the European Space Agency. Galileo is a GNSS that is under development.

3. GLONASS of Russia, created by the former Soviet Union and now operated by the Russian Aerospace Agency. It is similar to GPS in its architecture. GLONASS is an operational GNSS that is in the process of being modernized.

4. GPS of the United States. GPS is operational and in the process of being modernized.

5. IRNSS of India. India is developing the system for regional use.

6. QZSS of Japan. Japan is also developing a system. It complements GPS. It will be for regional usage.

GPS was the original GNSS system and is the dominant system. The impetus for developing new GNSSs is due to GPS’s success when used for positioning.

Receivers are available that determine position using signals from more than one GNSS.

GPS

In 1973 the Pentagon proposed a second-generation guidance, navigation, and positioning system, a global satellite navigation system. One of the reasons for the proposal was the absence of a system that a receiver could use at any time of the day. GPS was developed as part of a military satellite-based navigation system. The United States Department of Defense (DoD) wanted to use GPS as part of the NAVSTAR program for highly accurate navigation using radio-based ranging. As GPS is controlled and operated by the military, a number of its aspects are classified. The launching of satellites commenced in 1978, nearly 40 years ago. Initially, GPS was for military use but in 1983 the US President announced that it would be available for civilian use once completed. By the mid-1980s, GPS had evolved to the extent that it possessed many of its present day capabilities. GPS was partially operational by 1993 and fully operational by 1995. The Federal Radio Navigation Plan stipulated that GPS was to be the US Government’s main navigation system. The year 2015 saw the 20th anniversary of it being fully operational. Today, the network of satellites is called NAVSTAR–GPS (Navigation System Using Timing and Rangin–Global Positioning System).
The general public refers to it as GPS, whereas the military refers to it as NAVSTAR. It has become an accurate and stable long-term reference. GNSSs, such as the GPS, are currently the most accurate positioning systems available to navigators. GPS was quickly adopted by civilian users for a wide variety of positioning and navigation applications. Today, many millions of devices, down to smartphones, use GPS navigation. No charge is levied for making use of the satellites’ signals.

**Context and Applications**

Satellite navigation systems have numerous civilian uses, such as:

1. Farming.
2. Navigation:
   a. Walking—using hand-held devices. GPS is a familiar tool for backpackers. Suitable GPS-enabled devices are inexpensive and their functionality provides a huge supplement to that of a compass.
   b. Driving and other transportation uses—using devices installed in aircraft, cars, trucks, and ships (see Fig. 1). Civil aviation is rapidly adopting GPS utilization due to the advantages it possesses over other means of navigation.
   c. Emergencies—search and rescue.
3. Surveying.
4. Location-based services (LBSs).
5. Map making and the provision of data to a geographic information system (GIS).
6. Gathering sports data.
7. As a clock and to perform synchronization.
8. Geophysical sciences.

![Fig. 1 Use of GPS in transportation.](image-url)
10. Munitions—smart bombs or precision-guided munitions.

The applications evolved at a rapid rate.

As an example, farm tractors attached to a seed drill are available that allows a farmer to plant seeds accurately in a field. The farmer can position the implements to within 4 inches. As regards the user interface of the receiver in the tractor cab, the farmer could have a perspective view of the rows where he/she is intending to sow the seeds. The row in which sowing is currently taking place could be highlighted and the position of the tractor on that row shown. Around this visual display could be a variety of buttons for functions relevant to this application.

SURVEYING

Surveyors and engineers routinely use satellite surveying systems on site. GPS receivers used by surveyors are more sophisticated than the handheld ones used by the general public. Surveyors’ receivers are usually pole-mounted. A position is displayed to an accuracy of anything up to a few centimeters, or better. For less-demanding surveys, a low-order rover survey receiver could be used, giving submeter accuracy. For a construction site, a high-order roving receiver could be used, giving centimeter or millimeter accuracy. A GPS antenna can be mounted on an adjustable-length pole, a bipod, or a tripod. The vertical height (antenna reference height (ARH)) is calculated using Pythagoras’ theorem:

$$\text{ARH} = \sqrt{\text{SlantHeight}^2 - \text{AntennaRadius}^2}$$

A place where the coordinates are to be determined is called a new station. For each new station, all pertinent information is recorded (see Fig. 2). This includes the equipment numbers, the operator, the project number, the session times, and so on. There are different surveying techniques: traditional static, rapid static, reoccupation, kinematic surveying, and real-time kinematic (RTK) surveying.

The coordinates of relevant points can be uploaded from computer files prior to the surveyor going out to the field. When in the field, the surveyor’s initial task is to position the antenna pole over the point whose coordinates are to be determined. ARH is then measured, entered into a field log, and entered into the receiver. The accuracy of position at the location is displayed on the receiver.
GPS can be used in many different areas. An example area is open-pit mines, where quantities of material are to be determined. Another example area is deformation studies. This could involve geology, such as the movement of tectonic plates. It could also involve monitoring the stability of bridges and dams. Yet another example area is aerial surveying of land and water. A GPS receiver is located in an aircraft and is in contact with ground-based, or water-based, stations. When used with an Inertial Navigation System (INS), there is no need for external stations.

When changes to the land are being made, the engineer needs to know the existing height of the ground at a point and also what the proposed height is to be. GPS receivers can be standalone or can be integrated with other equipment, such as an inertial measurement unit (IMU). Portable equipment might come in a backpack or be handheld. When building structures, millimeter accuracy of vertical dimensions may be required and in this case GPS receivers can be integrated with laser devices.

Let us look at RTK in more detail. It makes use of differential positioning. It combines GPS receivers, mobile data communications, on-board applications, and on-board data processing. Its advent led to a new era in surveying. RTK makes use of a receiver at a base station and a roving receiver carried by a surveyor. Both receivers simultaneously track the same satellites. In addition, the satellite signals received at the base station are retransmitted to the roving receiver. Fig. 3 shows the arrangement.
Base station GPS receiver/antenna receives satellite signals and hands them to a base station radio transmitter that broadcasts them.

Surveyor carries a GPS antenna (for receiving the satellites’ signals).

Surveyor also carries a backpack containing a receiver connected to the antenna, as well as a radio receiver and radio (for receiving the base station’s signals).

Fig. 3 RTK surveying.

Communication between the base station and the surveyor can be facilitated with the use of cell phones. At the start of the surveying session, it is not necessary for the surveyor to know the vector between the base station and his/her position (called the baseline). The surveyor carries an adjustable length pole on top of which is mounted an antenna and a data collector. The surveyor wears a backpack which contains a receiver, for receiving the satellites’ signals, and a radio and radio antenna, for receiving signals from the base station. The RTK technique has benefitted from significant technological advances and this gained the technique acceptance among surveyors.

The data collector (a.k.a. survey controller) used in an RTK survey eases the surveyor’s job. It has a number of applications, for example, cut and fill.

LOCATION-BASED SERVICES

Those who enjoy walking can use a handheld GPS receiver to find their location on a map; motorists make use of dashboard-mounted devices, of which there are a number of versions.

A main area of application of positioning is in transportation. Some of the so-called intelligent transportation systems (ITSs) make use of GPS. ITSs can be used in assisting those with broken-down vehicles,
in monitoring the location of cargo, in routing, in the management of accidents, and so on. Ships use GPS receivers. Apart from transportation, handling emergencies is an important application of positioning. When using the 112 emergency telephone number, an EU directive requires mobile phone operators to provide the emergency services with details of the location of the user, if they know it. Similarly, in the United States, when a call to 911 is made, the US Federal Communications Commission requires mobile operators to relay location information to the emergency services. The commercial sector has become greatly interested in LBSs. LBSs are available on mobile devices. LBSs can be used in the open and also inside premises. Examples of LBSs include tracking of static resources, using RF tags, and tracking of people or things that are moving. Unfortunately, GNSS cannot be used for indoor positioning. This is because GNSS requires line of sight with satellites as the signals are attenuated by solid objects. Solid objects also cause multipath. An indoor positioning system can make use of one of a number of wireless technologies, such as UWB. The existence of LBSs means that marketing and advertising strategies need to be rethought so as to make them local to the consumer. Organizations can track their off-site employees, who will be using a navigation aid. The ease with which a person or vehicle can be positioned has opened up the possibility of location-specific billing. **Fig. 4** shows a schematic that gives the basic interaction between a user and a context-sensitive service provider.
In future, GPS will be usable in harsher environments, such as within buildings and in tunnels. This brings with it a whole raft of new applications such as tracking of persons (within a building for security purposes, hospital patients, and fire-fighters) and locating assets (finding an item of medical equipment in a hospital). A system which tracks a person or locates an asset in real-time is termed a real-time location system (RTLS). It is predicted that there will be tremendous growth in RTLSs.

MAP MAKING

In the modern age, people are increasingly becoming dependent on road maps; consider the popularity of Google Maps, for example. The manual upkeep of these maps is prohibitively time-consuming. Maps need to be accurate and this means that they must be response to changes to roads. Research has been undertaken to use GPS data, and inference, to automatically update maps. This data could be taken from users’ GPS receivers.

To date, the research has focused on correcting the geometry of roads. However, another area of difficulty concerns road intersections. Problems include such things as roads missing, no entry roads unmarked, and roads closed. Road intersection problems occur more frequently than do geometry problems. However, the former problems are more difficult to infer than the latter. Research has been undertaken in Shanghai to automatically update road intersection information (Wang, Forman, & Wei, 2015). Over a period of 21 months, the GPS data from more than 10,000 taxis was processed.

The Shanghai research (Wang et al., 2015) involved the use of an established algorithm. Simple algorithms were then proposed for identifying errors in maps—roads missing, no entry roads unmarked, and roads closed.

SPORTS DATA

One area involves gathering big game data, that is, soccer data, and using a program to analyze this data. Players wear GPS receivers and data is collected during a match. The wearing of GPS units is not allowed by some governing authorities but may be acceptable at lower levels, all the way down to training sessions. Professional football clubs employ Performance Analysts whose job is to use such a program. An analyst’s job is to get
simple statistical analyses in order to help players, coaches, and soccer club owners.

Companies developing such software have existed since the late 1990s. They develop technology that exploits the potential for gathering data, and analyzing it, in order to assist those making decisions in professional soccer. An example company is Prozone. A program could provide support to a club at the various levels: first team, reserves, and those in any academy. At each level one would expect a different level of performance. Over a period of time a club collects historical data and this can be used to benchmark players. A club can then compare the statistics for an individual player with existing and previous players.

GPS data can supplement data from other sources such as heart rate monitors. More generally, an overview of a player can be assembled from his/her health monitoring data, medical history, salary history, and income above and beyond his/her salary.

In rugby, players use wearable technology—GPS receivers in the back of their shirts.

A cyclist could use a GPS receiver to gather data about himself/herself. The data is uploaded into either a mobile or online app. The purpose of the app is to help the cyclist see how well (or how badly) he/she is performing. Thanks to the Tour de France and other events, as well as the realization that bikes are a healthy form of transport, cycling appears to be gaining in popularity. Example groups of cyclists are:
1. professional cyclists,
2. middle aged men in lycra (MAMILs), etc.

A relevant app to this application is Strava. A cyclist can compare his/her performance with that of other cyclists. A cyclist can upload GPS data to confirm that he/she took a certain period of time to complete a specific route. The cyclist can then check a league table to see how he/she ranks, compared to others, for cycling that route.

Runners make use of GPS-enabled watches connected to sensors in their shoes. After a run, a runner could upload the gathered GPS data into a program that shows the run on a map, indicating inclines.

**UNCERTAINTY IN GPS POSITIONING**

If the position calculated by a GPS receiver is erroneous, too inaccurate, or not available then this could have serious repercussions. Fortunately, over its long history, GPS has usually proved very successful. GPS receivers are
normally reliable, working correctly for many years without maintenance. There have been satellite malfunctions and cases of signal interference. Records for the deliberate interference of GPS signals are, obviously, difficult to come by. There have been many cases of unintentional interference. One case occurred in December 2001 at Moss Landing, California. A jammer was unintentionally left on causing GPS failure within a 180 nautical mile radius of Mesa, Arizona. A handheld jamming device (a.k.a. a personal privacy device (PPD)) can block GPS and mobile phone signals within a 20 meter radius. When GPS signals are unavailable, a GPS receiver goes into mode where it tries to predict the receiver’s position.

Automobile drivers frequently encounter problems due to car navigational equipment. A GPS receiver in an automobile comprises two parts, a system for processing satellite signals and a map system.

**GNSSs USAGE PATTERNS**

User equipment is in a continual stage of development. Most receivers are either built into mobile phones or located in automobiles.

**NONPOSITIONING USES OF GPS**

A receiver can calculate its position, its velocity, and the true time to a high accuracy.

Some users are only interested in the clock time broadcast by the satellites. We must not forget these users. They use the time information in their specific areas of work and are often dependent on the accuracy of the broadcasted times.

Let us turn our attention to how GPS satellite signals are used in time and frequency metrology. Laboratories can measure time in units of nanoseconds or smaller. In contrast, much larger units of time are enough for everyday use. In those industries where accurate timing is essential, the unit of time used is somewhere between those used in laboratories and those used in everyday life; it is the microsecond ($10^{-6}$ s). It is difficult for a human to comprehend how short a period of time a microsecond is and yet, as was mentioned, for a time metrologist, it is not that small. The critical-infrastructure applications the electric power grid and mobile telephone networks each need microsecond accuracy at thousands of geographically dispersed sites. They achieve this using GPS receivers that are used only as clocks, that is, GPS-enabled clocks. Currently, GPS is the only technology
that can provide microsecond accuracy at thousands of geographically dispersed sites.

GPS is trusted as a time reference because the clocks on the satellites are highly accurate atomic clocks whose time is controlled by the United States Naval Observatory (USNO). The time shown on a GPS-enabled clock has small inaccuracies due to a number of factors. Even in the worst case, the time on a GPS-enabled clock differs from Coordinated Universal Time (UTC) by no more than 0.4 $\mu$s.

The advent of GPS has enabled critical infrastructure technologies. These are technologies that heavily rely on GPS-enabled clocks because microsecond accuracy is easy to achieve with GPS but difficult to achieve without it. Two such technologies are code division multiple access (CDMA) mobile phone networks and the smart grid.

Mobile phone operators such as Nextel, Sprint, US Cellular, Verizon, and others make use of a type of network called a CDMA mobile phone network. If one looks in the vicinity of mobile phone antennas, perhaps atop a street light pole, one can often see a GPS antenna. The fixed base stations in a CDMA mobile phone network receive and transmit signals. When transmitting, a specific base station’s signal can be distinguished from other base station’s signal due to the fact that each base station makes use of a unique time offset when constructing its signal. GPS provides the time reference to which all base stations are synchronized. As a mobile phone moves from cell to cell, the handover of the phone’s signal from one base station to another is thus facilitated.

The electric power grid is subjected to considerable, and growing, demand from consumers, so much so that sometimes demand is close to what it is possible to supply. To prevent outages, the power grid can be made into a smart grid. In a smart grid, the state of the grid is monitored in real time. This is done by taking measurements, in a synchronized manner, at the power substations. GPS satellites provide this synchronization.

There are a number of possible backup strategies for GPS-enabled clocks. One is to use another GNSS; another is to resurrect eLoran, a radio navigation system that was shut down. Another possibility is to use the timing signals transmitted by networks. Using fiber optics, subnanosecond accuracy can be achieved. There are various ways, and complexities, with which timing is achieved in a network. However, all calculate the transmission delay between two clocks by sending a signal from a reference clock to a remote clock; the remote clock then returns the signal, and the reference clock notes the delay in the two-hop trip. Halving this delay is
an estimate of the transmission delay of a signal sent from a reference clock to a remote clock. The process is illustrated in Fig. 5. The remote clock is corrected whenever the reference clock synchronizes it. As a backup for GPS-enabled clocks used in a critical infrastructure application, the possible network solutions are:

1. Building a wide area network (WAN) based on fiber optics whose sole purpose was timing.
2. Building a WAN based on fiber optics whose timing was closely controlled.
3. Deploying large numbers of reference clocks, so that any remote clock is a short distance away from a local area network (LAN).

Yet another backup strategy is to deploy many thousands of atomic clocks. A final backup strategy will now be discussed. A satellite other than a GPS satellite could be used. The on-ground reference and remote clocks both receive the same satellite signal. Both clocks are calibrated to take account of the different signal delays. The time of receipt of the signal by the reference clock is passed via a network to a server as is the time of the receipt by the remote clock. The server calculates the difference in times and sends this correction to the remote clock. The above process can be performed continuously. The remote clock is referred to as a common-view disciplined clock (CVDC). For each CVDC only a small amount of data needs to be processed by the server. As a result the server can handle many CVDCs. CVDC systems exist in Japan and the United States. A fail-safe system could be established whereby each component of the system—the common-view signal, the data network, and the reference clock—each has a backup. The backup for the common-view signal could come from a GNSS satellite or a geocommunication satellite. Multiple data networks could be used so as to provide redundancy. Whenever one of the primary components of the CVDC system fails, the backup component is brought into action.
Time and Frequency Measurements

**GPS Receivers**

There are a number of different types of GPS receiver used in time and frequency metrology. When used for this purpose a receiver is referred to as a GPS timing receiver. There are innumerable time and frequency applications. Most receivers provide a 1 pps output and time-of-day information. Another type of GPS receiver also provides standard frequencies. These are called GPS disciplined oscillators (GPSDOs). They have many applications. For some of the more specialized measurements, two other types of GPS receiver are used. One is a common-view GPS receiver, the other is a carrier-phase GPS receiver. The latter type is designed for geodetic and surveying applications. The GPS antennas used with most receivers, in metrology, are small—usually <100 mm in diameter.

**Measurement Techniques**

As stated above, different types of receiver are used in time and frequency metrology. Consequently, different types of GPS measurements are made. The measurement techniques are one-way, single-channel common-view, multichannel common-view, and carrier-phase common-view. This list is ordered, starting from the technique with the most timing, and frequency, uncertainties to the least uncertainties. The frequency uncertainty for the one-way technique, for 1 day, is $< 2 \times 10^{-13}$.

**One-Way**

The one-way GPS measurement technique uses the signals output from a GPS receiver as a reference for calibration purposes. As with all receivers, prior to use, signal acquisition must be performed. Once this is completed, the output signal(s) can be input to the measurement system in use.

GPS satellites transmit signals that have been designed to be in close agreement with UTC and so the long-term accuracy of a GPS receiver is excellent. The only time when this was not the case was when the DoD deliberately added noise to GPS signals in order to reduce positioning and timing accuracy. This practice was called selective availability (SA). It ceased on May 2, 2000.

When a laboratory is making a measurement, it must know what the uncertainty of the measurement process is with respect to a reference. The uncertainty of the reference must be known with respect to a superior reference, etc., all the way back to the International System of Units (SI)
reference. Traceability requires an unbroken chain of comparisons with references, all comparisons having stated uncertainties. An example of a traceability train is:

SI → UTC (NIST) → UTC (USNO) → GPS Broadcast Signals →
GPS Received Signals → Users Device Being Tested by Laborator
where UTC (NIST) is the variant of UTC maintained by the National Institute of Standards and Technology and UTC (USNO) is the variant maintained by the United States Naval Observatory.

Another example traceability train is:

SI → UTC (NIST) → GPS Broadcast Signals → GPS Received Signals
→ Users Device Under Test

The uncertainties for the comparisons involving SI, UTC (NIST), UTC (USNO), and GPS Broadcast Signals are very small, and are of little relevance to most measurements. The three uncertainties can be found from published documents. In the second train, NIST records time and frequency offsets for each GPS satellite. This enables it to calculate the uncertainty of the UTC (NIST)-GPS Broadcast Signals. The uncertainty of the comparison between GPS Broadcast Signals and GPS Received Signals is receiver dependent. In order to derive this uncertainty, we must be able to state a specification that the receiver will meet or exceed when correctly operated. The uncertainty of the GPS Received Signals—Users Device Under Test link is the uncertainty in the calibration procedure being undertaken at the laboratory.

Common-View

The common-view technique is a simple way of comparing two clocks located in different places. Fig. 6 illustrates the approach. The technique removes measurement errors that are common to both locations. The time on the two clocks are compared. Similarly, the frequencies of the two clocks are compared. The technique has its pedigree in making comparisons of international time and frequency standards. There are two types of GPS common-view measurements: single-channel and multichannel.

Single-channel common-view needs a GPS receiver that is capable of reading a tracking schedule, that is, informing it when to start taking measurements and which satellite to track. Published tracking schedules are available.

Multichannel common-view does not use a tracking schedule. A receiver simply records data from all the satellites in view. This results in the collection of more data than when single-channel common-view is used.
A disadvantage of the common-view technique is that the data collected must be transferred between the two clocks’ owners, following which it must be processed. Hence it may take some time for the measurements results to be available. To overcome this delay, some years ago NIST developed a prototype system comprised of a multichannel common-view receiver integrated with an Internet-enabled personal computer. The system could send data to a web server as soon as measurements were made.

The common-view technique works best when the distance between clocks, called the baseline, is a few thousand kilometers or less. For two clocks located on the continental United States, the time uncertainty and the frequency uncertainty, over one day and at $2\sigma$, should be $<10$ ns and approximately $1 \times 10^{-13}$, respectively.

As regards measurement uncertainty, the common-view technique is only slightly better than the one-way technique. However, as regards traceability, with the common-view technique, the chain between UTC (NIST) and Users Device Under Test is reduced to one link.

**Carrier-Phase Common-View**

This technique is mainly used for frequency measurement. A GPS satellite transmits two signals—at the L1 and L2 frequencies. Each signal is comprised of a carrier wave and superimposed on this is a pseudorandom noise code. With the techniques mentioned above, the receiver processes the code on the L1 frequency, whereas with the carrier-phase approach, the receiver processes the carrier waves on both the L1 and L2 frequencies. The carrier-phase GPS technique necessitates substantial postprocessing of the data and so is not suited to everyday measurements. International
comparisons of frequency standards, however, make use of the technique. As mentioned above, two receivers are used in the common-view approach. For international comparisons where the baseline is prohibitively long, a network of receivers is used. The data collected is processed using precise satellite orbital information and detailed models of the ionosphere and troposphere. The equation used in the carrier-phase technique is:

$$\lambda \phi^S_k R = \sqrt{(x_k - x_0)^2 + (y_k - y_0)^2 + (z_k - z_0)^2 + c \tau + c \delta t_s + c \delta t_{eph} + c \delta t_{ion} + c \delta t_{trp} + \text{OtherErrors} + \epsilon}$$

where $c$ is the speed of light, $\lambda$ is the carrier wavelength, $c/f$, $\phi^S_k R$ is the carrier phase observable for satellite $S_k$ and receiver $R$, $(x_k, y_k, z_k)$ are the position of satellite $S_k$ when the data was sent, $(x_0, y_0, z_0)$ are the coordinates of the user receiver, $\tau$ is the receiver clock bias, $\delta t_s$ is the satellite clock error, $\delta t_{eph}$ is the ephemeris error, $\delta t_{ion}$ is the ionospheric error, and $\delta t_{trp}$ is the tropospheric error.

The other errors are ones such as multipath and noise. Research at NIST has shown that measurement uncertainty can be reduced by reducing the noise at both of the stations making the international comparison, by improving cycle slip detection, and by using good models of the ionosphere and troposphere.

In conclusion, GPS is the primary system for distributing highly accurate time and frequency worldwide.

**Further Information**

Appendix A discusses how one might calibrate a GPS receiver that is solely used for its timing pulse. A Regional Metrology Organization (RMO) called the Inter–American Metrology System, or Sistema Interamericano de Metrologia (SIM), created a comparison network to compare time and frequency standards between the member nations of SIM. This comparison network is described in Appendix B. SIM is made up of national metrology institutes in numerous countries throughout the Caribbean as well as Central, North, and South America. There are a small number of RMOs that are recognized by the Bureau International des Poids et Mesures (BIPM), one of which is SIM. Appendix C compares the characteristics of GPS receivers, as regards calibration of their delays, between several laboratories.

Furthermore, it may be possible to use a receiver as an acceleration sensor. Appendix D discusses this further.
ESTIMATING THE DISTANCE TO A GOLF FLAGSTICK

Two of the ways in which a golfer could estimate the distance to a flagstick are by using GPS and by using a laser device. Zhu & Vonderohe (n.d.) discusses the accuracies of the two approaches.

GPS

GPS does not calculate the distance between golf ball and flagstick directly. GPS can estimate the coordinates of a point on Earth by a receiver processing radio signals sent from several GPS satellites. A GPS receiver could estimate the coordinates of the golf ball and flagstick and then use them to calculate the distance between the points. The problem is that the golfer may not know the coordinates of the flagstick. The golfer could use an on-line map, possibly a satellite map, that gives the coordinates of any point selected. This will only suffice if the map is a high quality one and if the golfer is able to identify the flagstick on the map and point to it accurately. Let us analyze the errors in this approach. We would like to end up with a statement of the form “the error between the true distance and the measured distance is $d$ meters, 95% of the time.” The value of $d$ depends on how accurate the coordinates of the ball ($b$) can be measured 95% of the time, and how accurate the coordinates of the flagstick ($f$) can be measured 95% of the time. The following relationship holds (Zhu & Vonderohe, n.d.):

$$d = 0.8\sqrt{b^2 + f^2}$$

There are a number of problems that could cause major problems for GPS positioning. These include, among others: reflection of satellite signals off hard surfaces such as the walls of a building; sunspot activity, which disturbs the upper atmosphere; weak satellite signals. Furthermore, and as was mentioned, it may be very difficult to get the desired accuracy of the flagstick position due to the quality of the map used.

Laser Ranging

A laser ranging device can measure the distance from a golf ball to a flagstick, provided that the flagstick has a prism attached to it. It would be simpler to use than a GPS receiver requiring map interaction. A laser beam spreads out as it is transmitted. The accuracy of the device is proportional to the distance being measured.
Comparison Between GPS and Laser Ranging

A laser ranging device measures the distance between a golf ball and a flagstick directly, whereas a GPS receiver does not. When comparing the two technologies, it is important to be measure accuracy in a consistent way. For example, we could use a statement of the form “the error between the true distance and the measured distance is $x$ meters, 95% of the time” with both technologies. In conclusion, most professional golfers opt for laser ranging.

RECEIVER SPECIFICS

As there are widely different applications of GPS, there is a corresponding wide range of available receivers, differing in their functionality.

GPS receivers vary in price. The lowest precision receivers, used for day-to-day navigation, recreation (such as hiking and orienteering), and low-precision mapping/GIS are hand-held and cost from 100 to 500 dollars. Marine navigation receivers (accuracy 1–5 m) cost about 1000+ dollars. More precise receivers are used in GIS and mapping applications. With these, accuracy is improved with differential correction. For a receiver giving submeter to 5 m accuracy, the cost is between 1000 and 5000 dollars. For an accuracy of between 20 and 30 cm, the cost is between 5000 and 10,000 dollars. Receivers used in surveying have the highest accuracy (cm or mm accuracy) and cost from 5000 to 30,000 dollars.


The main characteristics of a receiver are the number of channels available to it (i.e., how many satellites it can track simultaneously), the number of frequencies it can receive (i.e., either L1 or both L1 and L2), and what part of a signal it uses to calculate the distance to a satellite (i.e., either a digital code or an analog carrier wave). Some GPS receivers use GLONASS as a backup.

AN INTERESTING ASIDE

Consider three researchers who wish to share an antenna that receives signals from a satellite. Each researcher has a hut full of equipment located somewhere on a large field. The signals are transferred from the antenna to
the huts by means of cables. The optimum position of the antenna, so as to minimize the length of cable used, is the Fermat-Torricelli point.

**FURTHER READING**


**REFERENCES**
